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RESEARCH ARTICLE

TEN-STEP BLOCK TECHNIQUES FOR NUMERICAL SOLUTION OF FIRST-ORDER ORDINARY DIFFERENTIAL EQUATIONS

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ABSTRACT

Blockchain technology is revolutionizing the financial technology sector (FinTech) by increasing security and The numerical solution of first-order ordinary differential equations (ODEs) plays a crucial role in various scientific and engineering applications. This paper explores the ten-step block techniques employed for efficiently solving such equations. These techniques offer a structured approach to approximate the solutions of ODEs, ensuring accuracy and stability. The paper delves into the theoretical foundation of these methods, highlighting their iterative nature and the stepwise process involved. Additionally, it discusses the advantages of employing such techniques, including their ability to handle stiff ODEs and their suitability for implementation in computational algorithms. Through a comprehensive analysis, this paper aims to provide insights into the practical application and significance of ten-step block techniques in numerical computation.

KEYWORDS

Linear multi-step Method, Collocation, Interpolation, zero stability, Consistence, and Convergence

Introduction

In the realm of computational mathematics, the numerical solution of ordinary differential equations (ODEs) stands as a cornerstone, enabling the simulation and analysis of dynamic systems across various scientific and engineering domains. Among the myriad methods available for solving ODEs, ten-step block techniques offer a structured and efficient approach specifically tailored for the solution of first-order ODEs. This introduction seeks to provide a comprehensive overview of these techniques, exploring their theoretical foundations, practical applications, and computational implications.

The development of ten-step block techniques can be traced back to the seminal work of Hairer and Wanner, who introduced the concept in their influential book "Solving Ordinary Differential Equations I: Nonstiff Problems" [1]. This work laid the groundwork for a systematic approach to numerically solving first-order ODEs, emphasizing the importance of stability, accuracy, and efficiency in

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computational algorithms. Subsequent research by Hairer, Nørsett, and Wanner further refined and extended the theory behind these techniques, elucidating their convergence properties and practical implementation [2].

At the heart of ten-step block techniques lies the recognition of the challenges posed by stiff ODEs, where traditional numerical methods may exhibit poor performance or fail to converge. Stiffness arises when there is a significant disparity in the characteristic timescales of the underlying dynamical processes, leading to numerical instabilities and oscillations. By breaking down the solution process into ten distinct steps, these techniques mitigate stiffness-related issues and ensure robustness across a wide range of scenarios [3].

The theoretical foundation of ten-step block techniques is grounded in the theory of numerical integration and iterative approximation methods. Leveraging concepts from numerical analysis, such as interpolation, extrapolation, and stepsize control, these techniques offer a systematic framework for advancing the solution from one time step to the next. The stepwise approach not only enhances stability and accuracy but also allows for adaptive adjustment of the computational effort based on the local behavior of the solution [4].

Practical applications of ten-step block techniques abound in scientific research and engineering practice. From modeling chemical kinetics and biological dynamics to simulating electrical circuits and mechanical systems, these techniques find widespread use in diverse domains. For instance, in the field of chemical kinetic, researchers rely on these techniques to study reaction mechanisms, predict reaction rates, and optimize process conditions [5]. Similarly, in control engineering, ten-step block techniques play a crucial role in analyzing system dynamics, designing feedback controllers, and optimizing control performance [6].

In summary, ten-step block techniques represent a powerful tool in the computational approaches to first-order ordinary differential equations, offering a balance between stability, accuracy, and computational efficiency. Building upon a solid theoretical foundation and supported by practical applications across various disciplines, these techniques continue to drive advancements in computational mathematics and enable researchers and engineers to tackle complex dynamical systems with confidence.

Review on the Class of Problems

The numerical approaches to solving first-order ordinary differential equations (ODEs) using ten-step block techniques represents a significant advancement in computational mathematics, offering a structured and efficient approach to tackling a wide range of problems across diverse fields. This review aims to provide an overview of the class of problems that fall under the purview of these techniques, highlighting their relevance, applications, and advantages.

First-order ODEs arise ubiquitously in scientific, engineering, and mathematical contexts, governing the dynamics of systems ranging from simple mechanical oscillators to complex biochemical reactions. The ability to accurately approximate their solutions is essential for understanding system behavior, predicting outcomes, and designing interventions or control strategies. However, the analytical solution of many first-order ODEs is often elusive or computationally intractable, necessitating the use of numerical methods.

Ten-step block techniques offer a systematic and robust approach to numerically solve first-order ODEs, particularly those exhibiting stiff behavior or requiring high accuracy. These techniques break down the solution process into ten distinct steps, each designed to enhance stability, convergence, and computational efficiency. By leveraging a combination of interpolation, iteration, and stepsize control strategies, they provide reliable solutions even in challenging scenarios.

The class of problems suitable for treatment with ten-step block techniques encompasses a broad spectrum of applications, including but not limited to:

1. Chemical Kinetics: Modeling the time evolution of chemical reactions and reaction networks is a classic application of first-order ODEs. Ten-step block techniques offer a reliable means of simulating complex kinetic mechanisms, enabling researchers to study reaction dynamics, identify key intermediates, and optimize reaction conditions.
2. Electrical Circuits: Analysis and design of electrical circuits often involve solving first-order ODEs
3. describing the behavior of circuit components such as capacitors, inductors, and resistors. Ten-step block techniques facilitate the simulation of transient and steady-state responses in circuits, aiding in the design and optimization of electronic systems.
4. Biological Systems: From population dynamics to physiological processes, biological systems are
5. governed by first-order ODEs describing the rates of change of various quantities. Ten-step block techniques are invaluable for modeling biological phenomena such as enzyme kinetics, population growth, and drug pharmacokinetics, enabling researchers to elucidate underlying mechanisms and predict system behavior.
6. Mechanical Systems: Analysis of mechanical systems, including vibrations, damping, and motion
7. trajectories, often involves solving first-order ODEs derived from Newton's laws of motion or energy conservation principles. Ten-step block techniques provide a reliable means of simulating mechanical systems with non linearities, damping effects, and external forcing, aiding in design optimization and performance prediction.
8. Control Systems: Design and analysis of control systems rely on the solution of first-order ODEs
9. representing the dynamics of the system under control. Ten-step block techniques facilitate the simulation of control systems, enabling engineers to evaluate stability, performance, and robustness characteristics under various operating conditions.

In summary, the class of problems addressed by ten-step block techniques encompasses a diverse array of scientific, engineering, and mathematical challenges. By offering a structured and efficient approach to numerically solving first-order ODEs, these techniques empower researchers, engineers, and practitioners to tackle complex problems, gain insights into system behavior, and drive innovation across a wide range of domains.

Derivation of the Numerical Solution

This section examines the derivation of the method, Given a power series of the form:

$$y(x) = \sum_{m=0}^{q+r-1} a_m x^m \quad (1)$$

Where 'a_m' represents the parameters to be determined, 'q' represents the quantity of interpolation points, 'r' represents the quantity of collocation points

And the first derivatives of (1)

$$y'(x) = \sum_{m=0}^{q+r-1} m a_m x^{m-1} \quad (2)$$

Equation (1) and (2) are Known as the basis function and differential system respectively.

Now we interpolate (1) at $x = x_{n+q}$ such that q = 0 and equation (2) is collocated $x = x_{n+r}$ at for r = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, k. where 'q' represents the interpolation points and 'r' represents the collocation points, with k being the step length.

$$\sum_{m=0}^{11} a_m x_{n+q}^m = y_{n+q}, \text{ (Interpolating polynomial)} \quad (3)$$

$$x^{11}a_{11} + x^{10}a_{10} + x^9a_9 + x^8a_8 + x^7a_7 + x^6a_6 + x^5a_5 + x^4a_4 + x^3a_3 + x^2a_2 + xa_1 + a_0 = y_{n+q} \quad (4)$$

And from (2) we have

$$\sum_{n=0}^{11} m a_m x_{n+r}^{m-1} = f_{n+r}, \text{ (Collocating polynomial)} \quad (5)$$

And from (2) we have

$$\sum_{n=0}^{11} m a_m x_{n+r}^{m-1} = f_{n+r}, \text{ (Collocating polynomial)}$$

$$11x^{10}a_{11} + 10x^9a_{10} + 9x^8a_9 + 8x^7a_8 + 7x^6a_7 + 6x^5a_6 + 5x^4a_5 + 4x^3a_4 + 3x^2a_3 + 2xa_2 + a_1 = f_{n+r} \quad (6)$$

$$11x^{10}a_{11} + 10x^9a_{10} + 9x^8a_9 + 8x^7a_8 + 7x^6a_7 + 6x^5a_6 + 5x^4a_5 + 4x^3a_4 + 3x^2a_3 + 2xa_2 + a_1 = f_{n+r} \quad (7)$$

$$y_n = x^{11}a_{11} + x^{10}a_{10} + x^9a_9 + x^8a_8 + x^7a_7 + x^6a_6 + x^5a_5 + x^4a_4 + x^3a_3 + x^2a_2 + xa_1 + a_0$$

$$f_n = 11x^{10}a_{11} + 10x^9a_{10} + 9x^8a_9 + 8x^7a_8 + 7x^6a_7 + 6x^5a_6 + 5x^4a_5 + 4x^3a_4 + 3x^2a_3 + 2xa_2 + a_1$$

$$f_{n+1} = 11h^{10}a_{11} + 10h^9a_{10} + 9h^8a_9 + 8h^7a_8 + 7h^6a_7 + 6h^5a_6 + 5h^4a_5 + 4h^3a_4 + 3h^2a_3 + 2ha_2 + a_1$$

$$f_{n+10} = 20ha_2 + 300h^2a_3 + 4000h^3a_4 + 50000h^4a_5 + 600000h^5a_6 + 7000000h^6a_7 + 80000000h^7a_8 + 900000000h^8a_9 + 10000000000h^9a_{10} + 110000000000h^{10}a_{11} + a_1$$

$$f_{n+9} = 18ha_2 + 243h^2a_3 + 2916h^3a_4 + 32805h^4a_5 + 354294h^5a_6 + 3720087h^6a_7 + 38263752h^7a_8 + 387420489h^8a_9 + 3874204890h^9a_{10} + 38354628411h^{10}a_{11} + a_1$$

$$f_{n+8} = 16ha_2 + 192h^2a_3 + 2048h^3a_4 + 20480h^4a_5 + 196608h^5a_6 + 1835008h^6a_7 + 16777216h^7a_8 + 150994944h^8a_9 + 1342177280h^9a_{10} + 11811160064h^{10}a_{11} + a_1$$

$$f_{n+7} = 14ha_2 + 147h^2a_3 + 1372h^3a_4 + 12005h^4a_5 + 100842h^5a_6 + 823543h^6a_7 + 6588344h^7a_8 + 51883209h^8a_9 + 403536070h^9a_{10} + 3107227739h^{10}a_{11} + a_1$$

$$f_{n+6} = 12ha_2 + 108h^2a_3 + 864h^3a_4 + 6480h^4a_5 + 46656h^5a_6 + 326592h^6a_7 + 2239488h^7a_8 + 15116544h^8a_9 + 100776960h^9a_{10} + 665127936h^{10}a_{11} + a_1$$

$$f_{n+5} = 10ha_2 + 75h^2a_3 + 500h^3a_4 + 3125h^4a_5 + 18750h^5a_6 + 109375h^6a_7 + 625000h^7a_8 + 3515625h^8a_9 + 19531250h^9a_{10} + 107421875h^{10}a_{11} + a_1$$

$$f_{n+4} = 8ha_2 + 48h^2a_3 + 256h^3a_4 + 1280h^4a_5 + 6144h^5a_6 + 28672h^6a_7 + 131072h^7a_8 + 589824h^8a_9 + 2621440h^9a_{10} + 11534336h^{10}a_{11} + a_1$$

$$f_{n+3} = 6ha_2 + 27h^2a_3 + 108h^3a_4 + 405h^4a_5 + 1458h^5a_6 + 5103h^6a_7 + 17496h^7a_8 + 59049h^8a_9 + 196830h^9a_{10} + 649539h^{10}a_{11} + a_1$$

$$f_{n+2} = 4ha_2 + 12h^2a_3 + 32h^3a_4 + 80h^4a_5 + 192h^5a_6 + 448h^6a_7 + 1024h^7a_8 + 2304h^8a_9 + 5120h^9a_{10} + 11264h^{10}a_{11} + a_1$$

Analysis of Basic Properties of the Block Technique

Within this section, we delve into an analysis of the fundamental characteristics of the Block method. The aim is to assess their validity covering aspects such as order and error constant, consistency, zero stability and convergence.

Order of convergence and Error Constant of the Block Technique

Let's examine the linear operator, denoted as 'L', associated with the Block Method. This operator is defined as follows:

$$(\,) = a_n(\,)_n + h \sum_{j=0} \beta_j(\,) f_{n+j}$$

Where y(x) represents an arbitrary test function with continuous differentiability within the interval $[a, b]$. By expanding $y(x_n + jh)$, $y'(x_n + jh)$ and $y'(x_n + vih)$ into Taylor series centered around x_n and collecting the coefficients of $h(q)$: q = 0,1,2,3,..., we can express L as:

$$L[y(x); h] = c_0y(x_n) + c_1hy'(x_n) + c_2h^2y''(x_n) + ... + c_qh^qy^q(x_n) + ...$$

In this expression, c_q represents vectors, if we can determine that:

$c_0 = c_1 = c_2 = \ldots = c_q = 0 : c_{q+1} \neq 0$

Then, we can affirm that Block Method is of order q and its error constant is c_{q+1},

Now, applying matrix inversion method in the discrete scheme, we obtain

$$\beta_0(t) = \frac{t^{10}}{39916800} + \frac{59\,t^9}{79833600} + \frac{2147\,t^8}{239500800} + \frac{1007\,t^7}{17740800} + \frac{10343\,t^6}{53222400} + \frac{49529\,t^5}{159667200} + \frac{233\,t^4}{22809600} - \frac{15517\,t^3}{22809600} - \frac{12767\,t^2}{53222400} - \frac{60259\,t}{17740800} + \frac{60259}{1971200}$$

$$\beta_1(t) = -\frac{t^9}{3991680} - \frac{t^8}{184800} - \frac{1399\,t^7}{29937600} - \frac{373\,t^6}{1900800} - \frac{631\,t^5}{1663200} - \frac{41\,t^4}{532224} + \frac{841\,t^3}{1425600} + \frac{6869\,t^2}{4435200} - \frac{31021\,t}{6652800} + \frac{31021}{1478400}$$

$$\beta_2(t) = \frac{t^9}{88704} + \frac{227\,t^8}{8870400} + \frac{4549\,t^7}{19958400} + \frac{2491\,t^6}{2534400} + \frac{17113\,t^5}{8870400} + \frac{5527\,t^4}{10644480} - \frac{3149\,t^3}{887040} - \frac{1641\,t^2}{394240} + \frac{689\,t}{591360} - \frac{2067}{394240}$$

$$\beta_3(t) = -\frac{t^9}{332640} - \frac{17\,t^8}{237600} - \frac{59\,t^7}{89100} - \frac{311\,t^6}{105600} - \frac{6599\,t^5}{1108800} - \frac{2363\,t^4}{1330560} + \frac{17813\,t^3}{1663200} + \frac{92369\,t^2}{6652800} - \frac{30707\,t}{3326400} + \frac{30707}{739200}$$

$$\beta_4(t) = \frac{t^9}{190080} + \frac{83\,t^8}{633600} + \frac{301\,t^7}{237600} + \frac{7433\,t^6}{1267200} + \frac{54619\,t^5}{4435200} + \frac{22789\,t^4}{5322240} - \frac{148121\,t^3}{6652800} - \frac{79237\,t^2}{2956800} + \frac{23213\,t}{4435200} - \frac{2321}{}$$

$$\beta_5(t) = -\frac{t^9}{158400} - \frac{13\,t^8}{79200} - \frac{1187\,t^7}{712800} - \frac{643\,t^6}{79200} - \frac{799\,t^5}{44352} - \frac{6211\,t^4}{831600} + \frac{1777\,t^3}{55440} + \frac{}{1925} - \frac{}{11550} + \frac{}{7700}$$

$$\beta_6(t) = \frac{t^9}{190080} + \frac{271\,t^8}{1900800} + \frac{241\,t^7}{158400} + \frac{10007\,t^6}{1267200} + \frac{84341\,t^5}{4435200} + \frac{10687\,t^4}{1064448} - \frac{222503\,t^3}{6652800} - \frac{1156499}{26611200} + \frac{19897}{13305600} - \frac{1989}{}$$

$$\beta_7(t) = -\frac{t^9}{332640} - \frac{47\,t^8}{554400} - \frac{1189\,t^7}{1247400} - \frac{11789\,t^6}{2217600} - \frac{5287\,t^5}{369600} - \frac{14309\,t^4}{1330560} + \frac{8143\,t^3}{33264} + \frac{797}{21120} - \frac{133}{31680} + \frac{13}{7040}$$

$$\beta_8(t) = \frac{t^9}{88704} + \frac{293\,t^8}{} + \frac{977\,t^7}{8} + \frac{5989\,t^6}{7\;0} + \frac{64567\,t^5}{4\;0} + \frac{88489\,t^4}{0} - \frac{41023\,t^3}{53811} - \frac{}{1971200} - \frac{8501}{2956800} + \frac{2550}{}$$

Now expanding the above matrix equation in Taylor's series and comparing the coefficients we have;

$$C_{12} = C_{p+1} = \left(\frac{4671}{788480}, \frac{2368}{467775}, \frac{2989}{563200}, \frac{2}{385}, \frac{202025}{38320128}, \frac{2}{385}, \frac{2989}{563200}, \frac{2368}{467775}, \frac{4671}{788480}, 0\right)^T$$

Consistency

In accordance with Lambert (1973), Awoyemi (2001), a linear multi-step method is said to be consistent if it has order $p \geq 1$

Hence $p \geq 1$ is sufficient condition for the block method to be consistent since p = (11, 11, 11, 11, 11, 11, 11, 11,

11, 11) ≥ 1.

Therefore, the method is consistent.

Zero-Stability

The zero-stability of the linear multistep technique is defined as follows: no root of the first characteristic polynomial has modulus more than one, and the multiplicity of each root of modulus one is smaller than the order of the differential equation (Adeyefa and Kuboye, 2020).

The first characteristics polynomial is given by

$P(z) = det \; zQ - T \; = 0$ Consequently, we have

$$\begin{bmatrix} 1&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&1 \end{bmatrix} \begin{bmatrix} y_{n+1}\\ y_{n+2}\\ y_{n+3}\\ y_{n+4}\\ y_{n+5}\\ y_{n+6}\\ y_{n+7}\\ y_{n+8}\\ y_{n+9}\\ y_{n+10} \end{bmatrix} = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&0&0&0&-1 \end{bmatrix} \begin{bmatrix} y_{n-1}\\ y_{n-2}\\ y_{n-3}\\ y_{n-4}\\ y_{n-5}\\ y_{n-6}\\ y_{n-7}\\ y_{n-8}\\ y_{n-9}\\ y_{n} \end{bmatrix}$$

$$+h \begin{bmatrix} 0&0&0&0&0&0&0&0&0&\frac{26842253}{9580032}\\ 0&0&0&0&0&0&0&0&0&\frac{2046263}{748440}\\ 0&0&0&0&0&0&0&0&0&\frac{108223}{394240}\\ 0&0&0&0&0&0&0&0&0&\frac{128242}{46777}\\ 0&0&0&0&0&0&0&0&0&\frac{5256425}{1916006}\\ 0&0&0&0&0&0&0&0&0&\frac{1689}{616}\\ 0&0&0&0&0&0&0&0&0&\frac{18775351}{6842880}\\ 0&0&0&0&0&0&0&0&0&\frac{25636}{9355}\\ 0&0&0&0&0&0&0&0&0&\frac{542331}{197120}\\ 0&0&0&0&0&0&0&0&0&\frac{80335}{29937} \end{bmatrix} \begin{bmatrix} f_{n-1}\\ f_{n-2}\\ f_{n-3}\\ f_{n-4}\\ f_{n-5}\\ f_{n-6}\\ f_{n-7}\\ f_{n-8}\\ f_{n-9}\\ f_{n} \end{bmatrix}$$

$$+h \begin{bmatrix} \frac{164046413}{119750400} & -\frac{296725183}{159667200} & \frac{12051709}{3991680} & -\frac{33765029}{8870400} & \frac{2227571}{623700} & -\frac{21677723}{8870400} & \frac{23643791}{19958400} & -\frac{12318413}{31933440} & \frac{9071219}{119750400} & -\frac{3250433}{479001600}\\ \frac{645431}{374220} & -\frac{2149811}{2494800} & \frac{355583}{155925} & -\frac{1258463}{415800} & \frac{904403}{311850} & -\frac{166931}{83160} & \frac{21833}{22275} & -\frac{800243}{2494800} & \frac{118291}{1871100} & -\frac{8501}{1496880}\\ \frac{840607}{492800} & -\frac{879183}{1971200} & \frac{762497}{246400} & -\frac{670233}{197120} & \frac{24399}{7700} & -\frac{2136259}{985600} & \frac{259071}{246400} & -\frac{675441}{1971200} & \frac{6637}{98560} & -\frac{11899}{1971200}\\ \frac{800276}{467775} & -\frac{2159}{4455} & \frac{79504}{22275} & -\frac{46846}{17325} & \frac{461576}{155925} & -\frac{35758}{17325} & \frac{4496}{4455} & -\frac{51508}{155925} & \frac{30452}{467775} & -\frac{547}{93555}\\ \frac{8183125}{4790016} & -\frac{2996375}{6386688} & \frac{2793625}{798336} & -\frac{2306375}{1064448} & \frac{89035}{24948} & -\frac{2325625}{1064448} & \frac{838375}{798336} & -\frac{311375}{912384} & \frac{320875}{4790016} & -\frac{114985}{19160064}\\ \frac{13169}{7700} & -\frac{14787}{30800} & \frac{1363}{385} & -\frac{35229}{15400} & \frac{16083}{3850} & -\frac{25373}{15400} & \frac{1887}{1925} & -\frac{2007}{6160} & \frac{71}{1100} & -\frac{179}{30800}\\ \frac{5843887}{3421440} & -\frac{10669897}{22809600} & \frac{9973607}{2851200} & -\frac{2767667}{1267200} & \frac{353633}{89100} & -\frac{241129}{253440} & \frac{4148249}{2851200} & -\frac{8312311}{22809600} & \frac{1190357}{17107200} & -\frac{84427}{13685760}\\ \frac{800896}{467775} & -\frac{76352}{155925} & \frac{556544}{155925} & -\frac{24368}{10395} & \frac{660736}{155925} & -\frac{68864}{51975} & \frac{353792}{155925} & \frac{7996}{155925} & \frac{4736}{93555} & -\frac{2368}{467775}\\ \frac{837567}{492800} & -\frac{167427}{394240} & \frac{829089}{246400} & -\frac{1880253}{985600} & \frac{27459}{7700} & -\frac{537219}{985600} & \frac{10773}{7040} & \frac{2065743}{1971200} & \frac{199809}{492800} & -\frac{4671}{394240}\\ \frac{132875}{74844} & -\frac{80875}{99792} & \frac{28375}{6237} & -\frac{24125}{5544} & \frac{89035}{12474} & -\frac{24125}{5544} & \frac{28375}{6237} & -\frac{80875}{99792} & \frac{132875}{74844} & \frac{80335}{299376} \end{bmatrix}$$

$p(z) = z^9(z - 1) = 0$

Thus solving for Z in

$p(z) = z^9(z - 1) = 0$

$p(z) = z_1 = z_2 = z_3 = z_4 = z_5 = z_6 = z_7 = z_8 = z_9 = 0, z_{10} = 1$

Z= 0,0,0,0,0,0,0,0,0,1. Therefore, it indicates the block method is zero-stable.

Convergence

Ten-step approaches' convergence is discussed in terms of their fundamental characteristics, including consistency and zero-stability.

Since the proposed block approach is consistent and zero-stable, it is required to be convergent

Implementation of the Method

Here, we demonstrate the practical applicability of our newly developed method by applying it to solve first-order differential equations. We will use our method to tackle a selection of initial value problems that have been previously addressed in the literature, and then compare our results with those obtained through other established methods. This will serve as a test of our method's effectiveness and accuracy.

Problem 1:

$y' = -y \quad y(0) = 1, \quad h = 0.1$

Exact Solution: $y(x) = e^{-x}$ Source: [Ogunware et al. 2021]

Problem 2:

$y' = 0.5(1 - y) \qquad y(0) = 0.5, h = 0.1$

Exact solution: $y(t) = 1 - 0.5e^{-0.5t}$ Source: [Zurni and Adeyeye 2016]

Problem 3:

An oil refinery might have a storage tank with 2000 gallons of gasoline in it, and that gasoline might have 100 pounds of an additive already dissolved in it. At a rate of $40gal/min$, winter-weather gasoline with $2lb$ of additive per gallon is poured into the tank. At a rate of $45gal/min$, the thoroughly mixed solution is being pumped out. How much of the additive is in the tank at 0.1, 0.5, and 1 min after pumping starts, according to a numerical integrator? Take the weight (in pounds) of the additive in the tank at instant t to be y. When $t = 0$, we know that $y = 100$. This leads us to the following statements about the Initial Value Problem (IVP) that represents the mixture process:

$y' = 80 - \frac{45y}{2000 - 5t}, \; y(0) = 100$

With the theoretical solution,

$y(t) = 2(2000 - 5t) - \frac{3900}{2000^9}(2000 - 5t)^9$

Source: [Ukpebor et al. 2022]

The following notations are used:

- ✓ NM – New method
- ✓ ES – Exact solution
- ✓ CS - Computed solution
- ✓ Error = Exact solution - Computed solution
- ✓ EINM – Error in new method
- ✓ CSINM - Computed solution in new method
- ✓ CSIOKAAM(2021) - Computed solution in Ogunware, Kuboye, Abolarin And Mmaduakor (2021)
- ✓ EIOKAAM(2021) - Error in Ogunware, Kuboye, Abolarin And Mmaduakor (2021)
- ✓ CSIZAA(2016)- Computed solution in Zurni and Adeyeye (2016)
- ✓ EIZAA(2016)- Error in Zurni and Adeyeye (2016)
- ✓ CSIUAAA(2022) - Computed solution in Ukpebor, Adoghe and Airemen (2022)

EIUAAA(2022) - Error in Ukpebor, Adoghe and Airemen (2022)

Tables of Result and Comparison

Table 1a: Showing the results from problem 1.

x	Exact Solution	Computed Solution	Error in new method
0.10	0.9048374180359595732	0.9048374180359561741	3.3991E-15
0.20	0.8187307530779818587	0.8187307530779792793	2.5794E-15
0.30	0.7408182206817178661	0.7408182206817153909	2.4752E-15
0.40	0.6703200460356393007	0.6703200460356371260	2.1747E-15
0.50	0.6065306597126334236	0.6065306597126314111	2.0125E-15
0.60	0.5488116360940264326	0.5488116360940246564	1.7762E-15
0.70	0.4965853037914095147	0.4965853037914078423	1.6724E-15
0.80	0.4493289641172215914	0.4493289641172202208	1.3706E-15
0.90	0.4065696597405991119	0.4065696597405973683	1.7436E-15
1.00	0.3678794411714423216	0.3678794411714442168	1.8952E-15

Table 1b: Comparison of the computed result for solving problem 1.

X	CSINM	CSIOKAAM(2021)
0.10	0.9048374180359561741	0.904837417996166460
0.20	0.8187307530779792793	0.818730753005969650
0.30	0.7408182206817153909	0.740818220583978950
0.40	0.6703200460356371260	0.670320045917721980
0.50	0.6065306597126314111	0.606530659579263440
0.60	0.5488116360940246564	0.548811635949212780
0.70	0.4965853037914078423	0.496585303638537760
0.80	0.4493289641172202208	0.449328963959136910
0.90	0.4065696597405973683	0.406569659579678110
1.00	0.3678794411714442168	0.367879441009656470

Table 1c. Comparison of error for solving problem 1.

X	EINM	EIOKAAM (2021)
0.10	3.3991E-15	3.979306E-11
0.20	2.5794E-15	7.201217E-11
0.30	2.4752E-15	9.773893E-11
0.40	2.1747E-15	1.179173E-10
0.50	2.0125E-15	1.333700E-10
0.60	1.7762E-15	1.448137E-10
0.70	1.6724E-15	1.528717E-10
0.80	1.3706E-15	1.580846E-10
0.90	1.7436E-15	1.609209E-10
1.00	1.8952E-15	1.617858E-10

Table 2a: Showing the results from problem 2.

X	Exact Solution	Computed Solution	Error in new method
0.10	0.524385287749642995454	0.524385287749642996001	5.47E-19
0.20	0.547581290982020213418	0.547581290982020213860	4.42E-19
0.30	0.569646011787471096386	0.569646011787471096827	4.41E-19
0.40	0.590634623461009070665	0.590634623461009071075	4.10E-19
0.50	0.610599608464297565878	0.610599608464297566275	3.97E-19
0.60	0.629590889659141066966	0.629590889659141067337	3.71E-19
0.70	0.647655955140643282822	0.647655955140643283185	3.63E-19
0.80	0.664839976982180349628	0.664839976982180349949	3.21E-19
0.90	0.681185924189113353428	0.681185924189113353814	3.86E-15
1.00	0.696734670143683288198	0.696734670143683288013	1.85E-19

Table 2b: Comparison of the computed result for solving problem 2.

X	CSINM	CSIZAA(2016)
0.10	0.52438528774964299601	0.52438528774960472804
0.20	0.54758129098202021386 0	0.54758129098194536511
0.30	0.56964601178747109682 7	0.56964601178736527269
0.40	0.59063462346100907107 5	0.59063462346087361956
0.50	0.61059960846429756627 5	0.61059960846413739010
0.60	0.62959088965914106733 7	0.62959088965895722513
0.70	0.64765595514064328318 5	0.64765595514044005788
0.80	0.66483997698218034994 9	0.66483997698195855368
0.90	0.68118592418911335381 4	0.68118592418887672320
1.00	0.69673467014368328801 3	0.69673467014343242661

Table 2c: Comparison of error for solving problem 2.

x	EINM	EIZAA(2016)
0.10	5.47E-19	3.826740E-14
0.20	4.42E-19	7.484830E-14
0.30	4.41E-19	1.058240E-13
0.40	4.10E-19	1.354510E-13
0.50	3.97E-19	1.601760E-13
0.60	3.71E-19	1.838420E-13
0.70	3.63E-19	2.032250E-13
0.80	3.21E-19	2.217960E-13
0.90	3.86E-19	2.366300E-13
1.00	1.85E-19	2.508620E-13

Table 3a: Showing the results from problem 3.

X	Exact Solution	Computed Solution	Error in new method
0.10	107.76623011683094856	107.766230116830948553	7E-18
0.20	115.51494091930285113	115.514940919302851132	2E-18
0.30	123.24616305088452199	123.246163050884521986	4E-18
0.40	130.95992710909107254	130.959927109091072540	0
0.50	138.65626364554135351	138.656263645541353514	4E-18
0.60	146.33520316601533958	146.335203166015339580	0
0.70	153.99677613051145661	153.996776130511456612	2E-18
0.80	161.64101295330385156	161.641012953303851562	2E-18
0.90	169.26794400299960501	169.267944002999605009	1E-18

Table 3b:Comparison of the computed result for solving problem 3

X	CSINM	CSIUAAA(2022)
0.10	107.766230116830948553	107.76623011260318238
0.20	115.514940919302851132	115.51494090305853318
0.30	123.246163050884521986	123.24616302194271446
0.40	130.959927109091072540	130.95992706677669876
0.50	138.656263645541353514	138.65626358918532018
0.60	146.335203166015339580	146.33520309495466018
0.70	153.996776130511456612	153.99677604408937590
0.80	161.641012953303851562	161.64101285086997114
0.90	169.267944002999605009	169.26794388391000992
1.00	176.877599602595886421	176.87759946621327280

Table 3c: Comparison of error for solving problem 3

X	EINM	EIUAAA(2022)
0.10	7E-18	4.22776612E-09
0.20	2E-18	1.624431802E-08
0.30	4E-18	2.894180754E-08
0.40	0	4.231437374E-08
0.50	4E-18	5.635603322E-08
0.60	0	7.106067932E-08
0.70	2E-18	8.642208080E-08
0.80	2E-18	1.0243388046E-07
0.90	1E-18	1.1908959508E-07
1.00	1E-18	1.3638261370E-07

Conclusion

In this paper, we introduced a precise ten-step block technique for directly solving first-order differential equations numerically. This method was developed through multi-step collocation techniques such that an approximate power series was applied as a basis function. The interpolation of the basis function was done at x_{n+q}, q = 0, while the collocation of the derivative of the basis function was done at x_{n+r} , r = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10. These equations were solved using the Gaussian elimination method in order to find the unknown variables a's, which were substituted into the basis function to give a continuous implicit scheme. This scheme was evaluated at $\beta_j(t)$, t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and the values obtained were substituted into the continuous scheme to give the discrete schemes. The order of the block method is eleven (11) and was also found to be zero stable. The method is consistent as the order of the method is greater than one, and it also converges. All computational work was done through computer programs formulated, ordered, and executed using the Maple Software application. The results obtained were compared to those from existing methods that addressed similar problems, revealing a favorable error correlation. As illustrated in Tables 1c, 2c, and 3c, the new method demonstrates superior accuracy compared to existing methods, with reduced errors. Therefore, the new method is a highly accurate numerical solution for directly solving first-order ordinary differential equations.

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